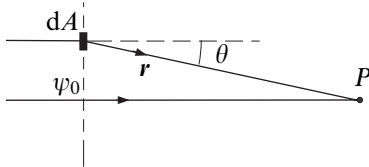
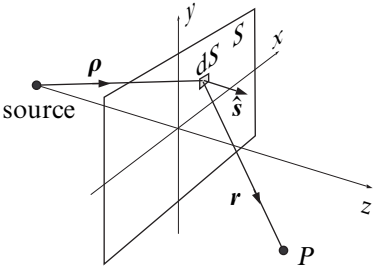


8.4 Fresnel diffraction

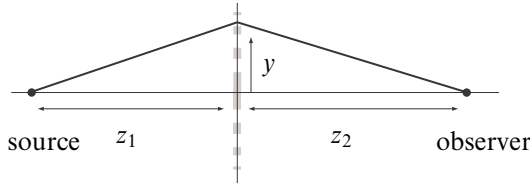
Kirchhoff's diffraction formula^a

 (source at infinity)		
Source at infinity where: Obliquity factor (cardioid)	$\psi_P = -\frac{i}{\lambda} \psi_0 \int_{\text{plane}} K(\theta) \frac{e^{ikr}}{r} dA \quad (8.45)$ $K(\theta) = \frac{1}{2}(1 + \cos\theta) \quad (8.46)$	ψ_P complex amplitude at P λ wavelength k wavenumber ($= 2\pi/\lambda$) ψ_0 incident amplitude θ obliquity angle r distance of dA from P ($\gg \lambda$) dA area element on incident wavefront K obliquity factor dS element of closed surface $\hat{s}$ unit vector $\hat{s}$ vector normal to dS $\hat{r}$ vector from P to dS $\hat{\rho}$ vector from source to dS E_0 amplitude (see footnote)
Source at finite distance ^b	$\psi_P = -\frac{iE_0}{\lambda} \oint_{\text{closed surface}} \frac{e^{ik(\rho+r)}}{2\rho r} [\cos(\hat{s} \cdot \hat{r}) - \cos(\hat{s} \cdot \hat{\rho})] dS \quad (8.47)$	

^aAlso known as the “Fresnel–Kirchhoff formula.” Diffraction by an obstacle coincident with the integration surface can be approximated by omitting that part of the surface from the integral.

^bThe source amplitude at ρ is $\psi(\rho) = E_0 e^{ik\rho}/\rho$. The integral is taken over a surface enclosing the point P .

Fresnel zones

		
Effective aperture distance ^a	$\frac{1}{z} = \frac{1}{z_1} + \frac{1}{z_2} \quad (8.48)$	z effective distance z_1 source–aperture distance z_2 aperture–observer distance
Half-period zone radius	$y_n = (n\lambda z)^{1/2} \quad (8.49)$	n half-period zone number λ wavelength y_n n th half-period zone radius
Axial zeros (circular aperture)	$z_m = \frac{R^2}{2m\lambda} \quad (8.50)$	z_m distance of m th zero from aperture R aperture radius

^aI.e., the aperture–observer distance to be employed when the source is not at infinity.

Cornu spiral

Fresnel integrals ^a	$C(w) = \int_0^w \cos \frac{\pi t^2}{2} dt \quad (8.51)$ $S(w) = \int_0^w \sin \frac{\pi t^2}{2} dt \quad (8.52)$	C Fresnel cosine integral S Fresnel sine integral
Cornu spiral	$CS(w) = C(w) + iS(w) \quad (8.53)$ $CS(\pm\infty) = \pm \frac{1}{2}(1 + i) \quad (8.54)$	CS Cornu spiral v, w length along spiral
Edge diffraction	$\psi_P = \frac{\psi_0}{2^{1/2}} [CS(w) + \frac{1}{2}(1 + i)] \quad (8.55)$ $\text{where } w = y \left(\frac{2}{\lambda z} \right)^{1/2} \quad (8.56)$	ψ_P complex amplitude at P ψ_0 unobstructed amplitude λ wavelength z distance of P from aperture plane [see (8.48)] y position of edge
Diffraction from a long slit ^b	$\psi_P = \frac{\psi_0}{2^{1/2}} [CS(w_2) - CS(w_1)] \quad (8.57)$ $\text{where } w_i = y_i \left(\frac{2}{\lambda z} \right)^{1/2} \quad (8.58)$	coherent plane waves
Diffraction from a rectangular aperture	$\psi_P = \frac{\psi_0}{2} [CS(v_2) - CS(v_1)] \times [CS(w_2) - CS(w_1)] \quad (8.59)$ $\text{where } v_i = x_i \left(\frac{2}{\lambda z} \right)^{1/2} \quad (8.60)$ $\text{and } w_i = y_i \left(\frac{2}{\lambda z} \right)^{1/2} \quad (8.61)$ $\text{and } w_i = y_i \left(\frac{2}{\lambda z} \right)^{1/2} \quad (8.62)$	x_i positions of slit sides y_i positions of slit top/bottom

^aSee also Equation (2.393) on page 45.

^bSlit long in x.

